4.1 RADIAN AND DEGREE MEASURE

What you should learn

- Describe angles.
- Use radian measure.
- Use degree measure.
- Use angles to model and solve real-life problems.

Why you should learn it

You can use angles to model and solve real-life problems. For instance, in Exercise 119 on page 291, you are asked to use angles to find the speed of a bicycle.

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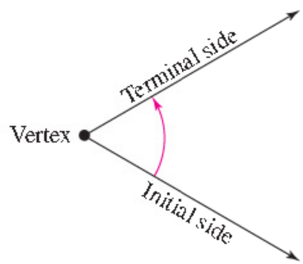
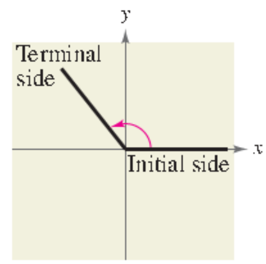
Angles

As derived from the Greek language, the word **trigonometry** means "measurement of triangles." Initially, trigonometry dealt with relationships among the sides and angles of triangles and was used in the development of astronomy, navigation, and surveying. With the development of calculus and the physical sciences in the 17th century, a different perspective arose—one that viewed the classic trigonometric relationships as *functions* with the set of real numbers as their domains. Consequently, the applications of trigonometry expanded to include a vast number of physical phenomena involving rotations and vibrations. These phenomena include sound waves, light rays, planetary orbits, vibrating strings, pendulums, and orbits of atomic particles.

The approach in this text incorporates *both* perspectives, starting with angles and their measure.



Angle
FIGURE 4.1



Angle in standard position
FIGURE 4.2

An **angle** is determined by rotating a ray (half-line) about its endpoint. The starting position of the ray is the **initial side** of the angle, and the position after rotation is the **terminal side,** as shown in Figure 4.1. The endpoint of the ray is the **vertex** of the angle. This perception of an angle fits a coordinate system in which the origin is the vertex and the initial side coincides with the positive x-axis. Such an angle is in **standard position,** as shown in Figure 4.2. **Positive angles** are generated by counterclockwise rotation, and **negative angles** by clockwise rotation, as shown in Figure 4.3. Angles are labeled with Greek letters α (alpha), β (beta), and θ (theta), as well as uppercase letters A, B, and C. In Figure 4.4, note that angles α and β have the same initial and terminal sides. Such angles are **coterminal.**

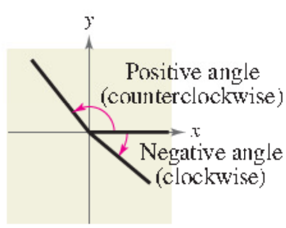


FIGURE 4.3

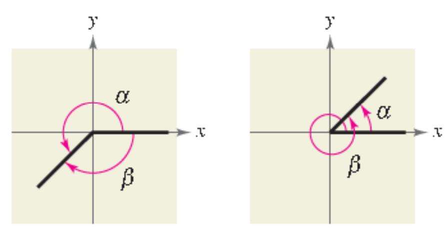


FIGURE 4.4 Coterminal angles

4.1 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

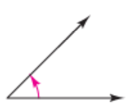
VOCABULARY: Fill in the blanks.

1. ________ means "measurement of triangles."
2. An ________ is determined by rotating a ray about its endpoint.
3. Two angles that have the same initial and terminal sides are ________.
4. One ________ is the measure of a central angle that intercepts an arc equal to the radius of the circle.
5. Angles that measure between 0 and $\pi/2$ are ________ angles, and angles that measure between $\pi/2$ and π are ________ angles.
6. Two positive angles that have a sum of $\pi/2$ are ________ angles, whereas two positive angles that have a sum of π are ________ angles.
7. The angle measure that is equivalent to a rotation of $\frac{1}{360}$ of a complete revolution about an angle's vertex is one ________.
8. 180 degrees = ________ radians.
9. The ________ speed of a particle is the ratio of arc length to time traveled, and the ________ speed of a particle is the ratio of central angle to time traveled.
10. The area A of a sector of a circle with radius r and central angle θ, where θ is measured in radians, is given by the formula ________.

SKILLS AND APPLICATIONS

In Exercises 11–16, estimate the angle to the nearest one-half radian.

11.



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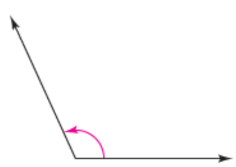

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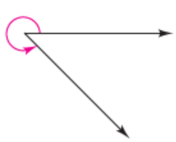

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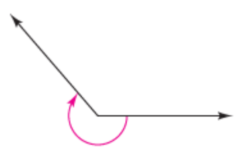


15.

16.

In Exercises 17–22, determine the quadrant in which each angle lies. (The angle measure is given in radians.)

17. (a) $\frac{\pi}{4}$ (b) $\frac{5\pi}{4}$

18. (a) $\frac{11\pi}{8}$ (b) $\frac{9\pi}{8}$

19. (a) $-\frac{\pi}{6}$ (b) $-\frac{\pi}{3}$

20. (a) $-\frac{5\pi}{6}$ (b) $-\frac{11\pi}{9}$

21. (a) 3.5 (b) 2.25

22. (a) 6.02 (b) -4.25

In Exercises 23–26, sketch each angle in standard position.

23. (a) $\frac{\pi}{3}$ (b) $-\frac{2\pi}{3}$

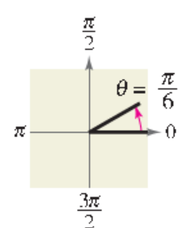
24. (a) $-\frac{7\pi}{4}$ (b) $\frac{5\pi}{2}$

25. (a) $\frac{11\pi}{6}$ (b) -3

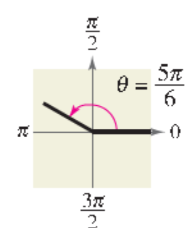
26. (a) 4 (b) 7π

In Exercises 27–30, determine two coterminal angles (one positive and one negative) for each angle. Give your answers in radians.

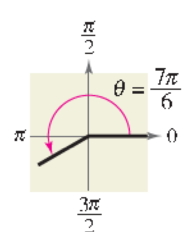
27. (a)



(b)



28. (a)



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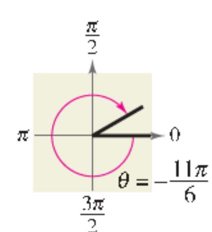


29. (a) $\theta = \frac{2\pi}{3}$ (b) $\theta = \frac{\pi}{12}$

30. (a) $\theta = -\frac{9\pi}{4}$ (b) $\theta = -\frac{2\pi}{15}$

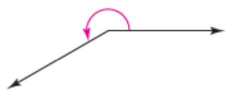
In Exercises 31–34, find (if possible) the complement and supplement of each angle.

31. (a) $\pi/3$ (b) $\pi/4$ **32.** (a) $\pi/12$ (b) $11\pi/12$

33. (a) 1 (b) 2 **34.** (a) 3 (b) 1.5

In Exercises 35–40, estimate the number of degrees in the angle. Use a protractor to check your answer.

35.



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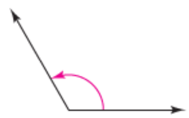

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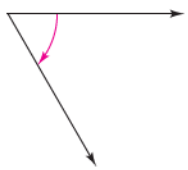

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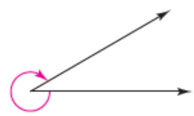


39.

40.

In Exercises 41–44, determine the quadrant in which each angle lies.

41. (a) 130° (b) 285°

42. (a) 8.3° (b) 257° 30′

43. (a) −132° 50′ (b) −336°

44. (a) −260° (b) −3.4°

In Exercises 45–48, sketch each angle in standard position.

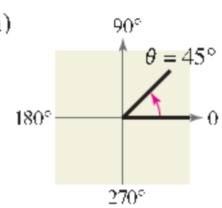
45. (a) 90° (b) 180° **46.** (a) 270° (b) 120°

47. (a) −30° (b) −135°

48. (a) −750° (b) −600°

In Exercises 49–52, determine two coterminal angles (one positive and one negative) for each angle. Give your answers in degrees.

49. (a)



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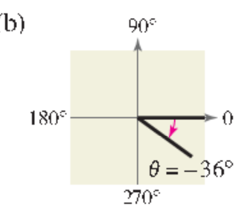
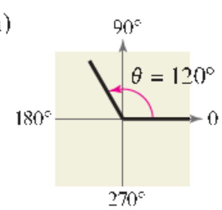


50. (a)



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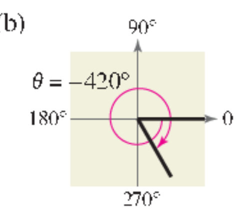

51. (a) $\theta = 240°$ (b) $\theta = -180°$

52. (a) $\theta = -390°$ (b) $\theta = 230°$

In Exercises 53–56, find (if possible) the complement and supplement of each angle.

53. (a) 18° (b) 85° **54.** (a) 46° (b) 93°

55. (a) 150° (b) 79° **56.** (a) 130° (b) 170°

In Exercises 57–60, rewrite each angle in radian measure as a multiple of π. (Do not use a calculator.)

57. (a) 30° (b) 45° **58.** (a) 315° (b) 120°

59. (a) −20° (b) −60° **60.** (a) −270° (b) 144°

In Exercises 61–64, rewrite each angle in degree measure. (Do not use a calculator.)

61. (a) $\frac{3\pi}{2}$ (b) $\frac{7\pi}{6}$ **62.** (a) $-\frac{7\pi}{12}$ (b) $\frac{\pi}{9}$

63. (a) $\frac{5\pi}{4}$ (b) $-\frac{7\pi}{3}$ **64.** (a) $\frac{11\pi}{6}$ (b) $\frac{34\pi}{15}$

In Exercises 65–72, convert the angle measure from degrees to radians. Round to three decimal places.

65. 45° **66.** 87.4°

67. −216.35° **68.** −48.27°

69. 532° **70.** 345°

71. −0.83° **72.** 0.54°

In Exercises 73–80, convert the angle measure from radians to degrees. Round to three decimal places.

73. $\pi/7$ **74.** $5\pi/11$

75. $15\pi/8$ **76.** $13\pi/2$

77. -4.2π **78.** 4.8π

79. −2 **80.** −0.57

In Exercises 81–84, convert each angle measure to decimal degree form without using a calculator. Then check your answers using a calculator.

81. (a) 54° 45′ (b) −128° 30′

82. (a) 245° 10′ (b) 2° 12′

83. (a) 85° 18′ 30″ (b) 330° 25″

84. (a) −135° 36″ (b) −408° 16′ 20″

In Exercises 85–88, convert each angle measure to degrees, minutes, and seconds without using a calculator. Then check your answers using a calculator.

85. (a) 240.6° (b) −145.8°

86. (a) −345.12° (b) 0.45°

87. (a) 2.5° (b) −3.58°

88. (a) −0.36° (b) 0.79°