Pre-Calculus **Name** ______________________________

Unit 2 Exponential and Logarithmic Functions Guiding Questions **Collected / Not Collected**

Objectives:

- Evaluate exponential and logarithmic function
- Identify key features and sketch the graphs of exponential and logarithmic functions
- Use exponential and logarithmic properties to solve equations
- Apply exponentials and logarithms to real world situations

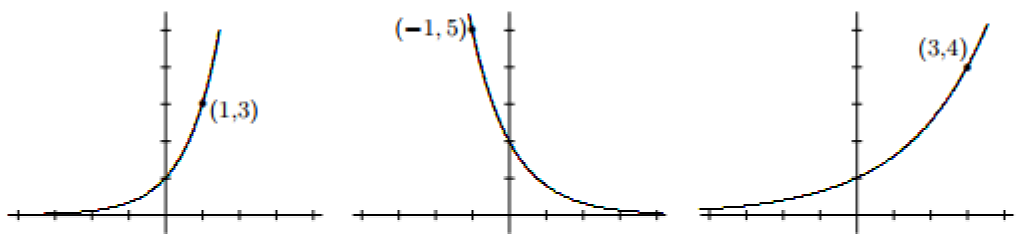
1. Many students find statements like $2^0 = 1$ and $2^{1/3} = \sqrt[3]{2}$ a bit mysterious, even though most of us have used them for years, so let's start there. Consider the list….

$$2^1, 2^2, 2^3, 2^4, 2^5 \ldots$$

a. What do you multiply by to get from a number on this list to the next number on the right?
b. Starting from any number on this list besides 2^1, what do you divide by to get from that number to the previous number on the left?
c. If we start at 2^1 and move to the left following this pattern, it suggests how we should define 2^0. What do you get for 2^0 if you follow this pattern?
d. If we now move from 2^0 another number to the left following the pattern, it suggests how we should define $2^{-1}, 2^{-2}, etc.$ What do you get for the values if you follow this pattern?
e. Does it matter for your reasoning that the base of the exponential was the number 2? Why or why not?

2. Graph the functions $f(x) = 2^x \; and \; g(x) = 2^{-x}$ and give the domain and range of each function.
a. Determine if each function is increasing or decreasing.
b. State the y-intercept, x-intercept, any asymptotes and 3 points on the graph of f(x), if possible.
c. Graph the inverse function of f(x). State the domain and range for the inverse.
d. State the y-intercept, x-intercept, any asymptotes and 3 points on the graph of the inverse, if possible.
e. Make some comments about your answers from parts a and d. What do you notice?

3. Find a plausible equation for each of the graphs below in the form of $y = a \cdot b^x$:



4. Show that it is possible to use logarithms to solve the equation $x^{2.5} = 1997$. Then show that it is not necessary to do so.

5. Simplify the equations by eliminating all references to logarithms.
(a) $0.5 \log y + \log x = \log 300$ **(b)** $1.5 \log y = 2.699 - \log x$

6. One approach to solving an equation like $5^x = 20$ is to calculate the base-10 logarithms of both sides of the equation. Justify the equation $x log = log 20$, then obtain the desired answer in the form $x = \frac{\log 20}{\log 5}$. Evaluate the expression. Notice that $log_5 20 = \frac{\log 20}{\log 5}$.

7. The function p is defined by $p(t) = 3960(1.02)^t$ describes the population of Dilcue, North Dakota t years after it was founded.
   a. Find the founding population.
   b. At what annual rate has the population of Dilcue been growing?
   c. Solve the equation when p(t) = 77218. What is the meaning of your answer?

8. Find the solutions to the following computational problems by using properties of exponentials and logarithms.

a. $10^{2x+1} = 100$

b. $2^{x^2} = 16$

c. $2^x = 4^{x+2}$

d. $Find\ log_2 8$

e. $Find\ ln(e^2)$

f. $e^{3x} = 3$

g. $4^x = e$

h. Solve $\ln(x+1) + \ln(x-1) = ln3$. Be sure to check your answers.

9. In summarizing the growth of a certain population Bailey writes $G(t) = 747t^{1.44}$ by mistake, instead of $G(t) = 747(1.44)^t$. Are there t-values for which the expressions agree in value?