Pre-Calculus **Name ________________________**

Unit 3 Rational Functions Guiding Questions **Collected/Not Collected**

Objectives:

- Graph a rational function by determining the key features of a graph (Domain, range, asymptotes, intercepts, and point discontinuities)
- Rewrite rational functions into simpler forms
- Perform operations on rational functions
- Solve problems using rational functions.

1. A rational function is a ratio of two polynomial functions, f(x) and g(x), which would be written as $\frac{f(x)}{g(x)}$.
   a. How would you determine any discontinuities for the rational function?
   b. Determine a rational function in which there are no discontinuities.
   c. Write a rational function that has discontinuities at x=0 and x=-3.

2. Determine the domain for the function $f(x) = \frac{x^4+3x^2-4}{x^2-1}$.

3. A question on a calculus test looks like this:
   Evaluate: $\lim_{x \to 1} \frac{x^2-1}{x-1}$ (Hint: this limit does not deal with asymptotes)
   a. While you may not know how to evaluate the limit, why does the question give the *hint*? Explain the hint in terms of rational functions and their properties.

4. Determine the values to place in the missing spots to solve the equation below. You may use integer values:
$$\frac{x^2+2x-8}{x^2+9x+20} \div \frac{x^2+[\quad]x+[\quad]}{x^2+[\quad]x+[\quad]} = \frac{x-1}{x+5}$$

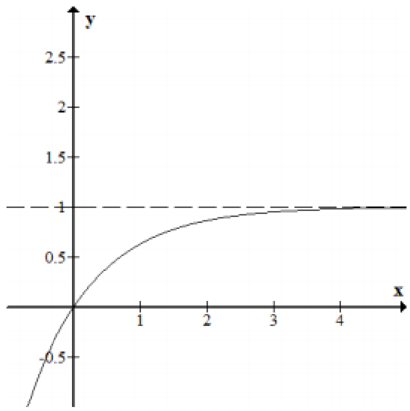
5. What numbers go in the blanks to make the equation true?
$$(2x^2+[\quad]x+3)([\quad]x+4) = 4x^3+20x^2+30x+12$$

6. Graph $f(x) = \frac{2x-3}{x+1}$, identify **all** asymptotic behavior. State the domain and the range of f.

7. Graph $f(x) = \frac{x^2-1}{x+1}$. What are the domain and range of f?
   a. What is the simplified form of f(x)?
   b. What about the graph of g(x) = x-1 is similar?
   c. What is different?
   d. Why can I not just enter the simplified form, g(x) = x-1, into my graphing calculator?

8. Horizontal asymptotes are an important part of rational functions. Notice the definition given,

Definition 2.2 *The line $y = b$ is a* **horizontal asymptote** *of the function $y = f(x)$ if y approaches b as x approaches $\pm\infty$.*



This graph has a horizontal asymptote at $x = 1$.

Let $f(x) = \frac{1}{x}$. Look at the f(x) values as your x values approach infinity.

x	$f(x)$
-10	-0.1
-100	-0.01
-1000	-0.001

x	$f(x)$
10	0.1
100	0.01
1000	0.001

As your x values get closer and closer to ±∞, what f(x) value do you appear to approach? (Try ±10,000 for your x value if you still aren't sure). From the definition, what does your horizontal asymptote appear to be for $f(x) = \frac{1}{x}$?

a. Try this same process with the function $h(x) = \dfrac{3x+5}{x+2}$ What is the value of your horizontal asymptote?

b. Rational functions are written in the form,

$$r(x) = \frac{ax+b}{cx+d} \text{ with } a, b, c, d \in \mathbb{R}$$

In order to find the horizontal asymptotes we will evaluate the function when x approaches ±∞. Because the values of x are getting so large, b and d are no longer significant. (Think about it, if a= 3 and b=6 and you evaluate. You would get 3,000,000 + 6. If you add 6 to 3,000,000, is 3,000,000 significantly different than 3,000,006?) Find the value of the horizontal asymptote by evaluating x values as they get closer to ±∞ for the function

$$r(x) = \frac{ax}{cx}$$

How would you describe the rule we could use for evaluating all rational functions that have the same degree for my numerator and denominator?

c. Find the horizontal asymptote for the rational function:

$$\frac{3x^2 - 2x - 1}{2x^2 + 3x - 2}$$

d. There are further rules for determining horizontal asymptotes for rational functions where the denominator has a larger degree than the numerator and vice versa. If time allows, make up a few rational functions and test them in your calculator to determine where the horizontal asymptotes would be in these situations.